Revolutionary polyhedron in "isonode" triangles

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Introduction

It was in the late 1960s that 3D parametric designs could be produced thanks to the FORTRAN IV language, provided a computer with the appropriate computing power and a graphic printer were available.

Consequently, I was able in 1970 to model the geometry of the "Wing Building" at the French-language Free University of Brussel (ULB)[1] computing centre, using this 3D virtual model to calculate its structure[2] the following year at the Massachusetts Institute of Technology (MIT).

However, the time available on the large computers needed for calculations, even at MIT, was limited and subject to quotas for everyone. So I had to decide to draw the graphical results by hand just as I studied the "isobaric" and "isonode" structures with a parallel T on a door leaf, acting as a drawing table, in my room at Harvard Street n°361 (apt 3 c/o Aragon), Cambridge, Massachusetts.

Upon my return to Brussels, in July 1972 all my energy was (and still is) devoted to learning the art of building, in parallel to the art of calculation, so I had to confine myself to displaying the meagre results of my reflections in the magazine "Neuf" in 1974, and then at the "Second International Conference on Space Structures" in 1975[3].

Hence 45 years later, in common with everyone else, I had all the computer power and technology I needed to resume my unfinished business. You do not write your own drawing or calculation programs anymore, you use them "ready-made". The following illustrations were produced by Ali Laghrari with Rhinoceros and his "assistant" Grasshopper.

Polyhedron generation

Figure 1 shows a circle of radius $\mathbf{R_0}$ circumscribing a regular polygon with **n** sides, each with a length of $\mathbf{2L_0}$ ($\mathbf{L_0}=\mathbf{R_0}.\sin(180°/\mathbf{n})$ where $\mathbf{L_0}=\mathbf{R_0}.\sin\mathbf{N}$; with $\mathbf{N}=180°/\mathbf{n}$). In the plane and within the polygon [4], these sides form the bases of **n isosceles triangles** "$\mathbf{2\alpha}$", with a height of $\mathbf{H_0}=\mathbf{R_0}.\sin\mathbf{N}/\tan\boldsymbol{\alpha}$ and a side $\mathbf{B_0}=\mathbf{R_0}.\sin\mathbf{N}/\sin\boldsymbol{\alpha}$ for an aperture angle at vertex $\mathbf{2\alpha}$. The polygon connecting these vertices is inscribed in a circle of radius $\mathbf{r_0}=\mathbf{R_0}.\sin(\alpha-\mathbf{N})/\sin\alpha$ and its **n** sides with a length of $\mathbf{2l_0}=2\mathbf{R_0}.\sin(\alpha-\mathbf{N}).\sin\mathbf{N}/\sin\alpha$ form the bases for a second series of **n isosceles triangles** with a vertex angle $\mathbf{2\beta}=2(\alpha-\mathrm{N})$ incorporated into the first series to form, together, a flat ring with 2n triangles.

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[1] "Structure du Wing Building", M.S. Thesis, Ph. Samyn, U.L.B., June 1971; (BE).

[2] "Analysis of a 40-storey building, reticulated membrane bearing wall", M.I.T., M.S. Thesis, Ph. Samyn, June 1972; (US).

[3] REVUE NEUF, n° 51, September – October 1974, pp. 53-58; (BE). Proceedings of the 2nd International Conference on Space Structures – University of Surrey, Guilford, England, 1975, pp. 621-634; (GB).

[4] They can also be placed, interchangeably, outside the polygon while remaining in its plane or perpendicular to the plane on the faces of a regular prism with n sides, to lead to the same result.

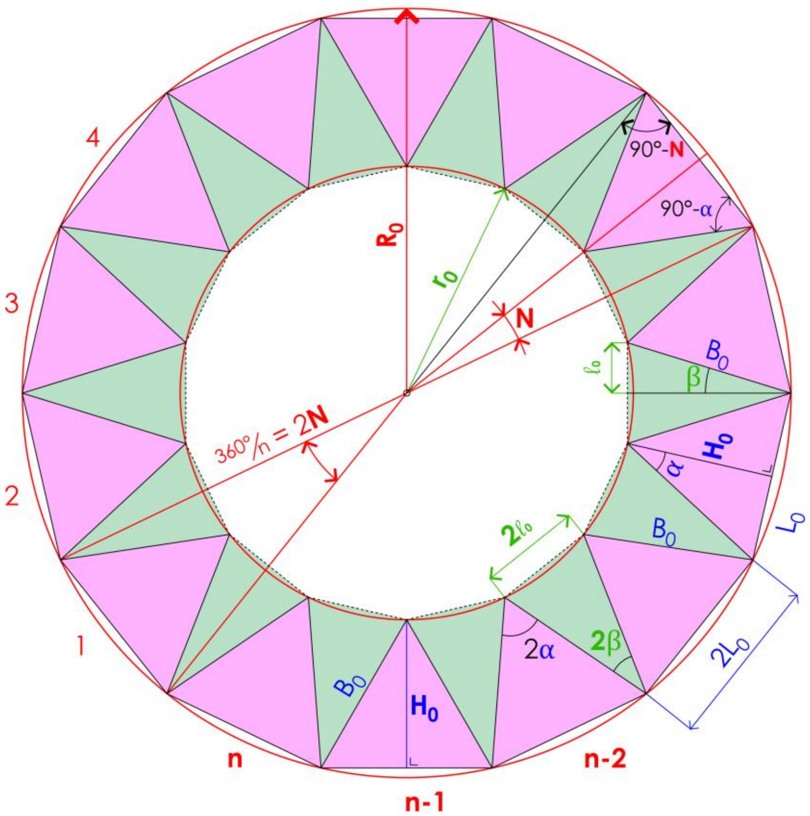
Figure 1

$$\mathbf{N} = \frac{180°}{\mathbf{n}}$$

$$L_0 = R_0 . \sin \mathbf{N}$$

$$H_0 = R_0 . \frac{\sin \mathbf{N}}{\tan \alpha}$$

$$B_0 = R_0 . \frac{\sin \mathbf{N}}{\sin \alpha}$$



$$2\beta = 2(\alpha - \mathbf{N})$$

$$r_0 = R_0 . \frac{\sin(\alpha - \mathbf{N})}{\sin \alpha}$$

$$l_0 = R_0 . \frac{\sin(\alpha - \mathbf{N}) . \sin \mathbf{N}}{\sin \alpha}$$

Figure 2 next illustrates the n triangles "**2α**" which rotate out of their plane by a γ angle around their bases to form the **first part** of the first series of isosceles triangles of the polyhedron. The polygon connecting their vertices is written in a circle of radius $R_1 = R_0.(1 - \sin \mathbf{N} . \cos\gamma / \tan\alpha)$ and its **n** sides with a length of $2L_1 = 2R_1 . \sin \mathbf{N}$ form the bases of the n triangles with an angle $2\beta_0$ at their vertex of the **second part** of the first series of the 2n isosceles triangles of the polyhedron. This first series consisting of n triangles **2α** and n triangles $2\beta_0$ is inscribed in a first frustoconical ring with an angle at the vertex of ($180° - 2\delta_0$).

A second set of n isosceles triangles "**2α**" with a $2L_1$ base is created inside the n-sided $2L_1$ polygon inscribed in the circle of radius R_1 and in its plane. These triangles rotate at an angle of 2γ around their base to form the **first part** of the second series of polyhedron triangles, with their vertices being connected to form a polygon inscribed in a circle of radius R_2 whose sides form the bases of the n triangles with an angle at the vertex $2\beta_1$ of the **second part** of the second series of triangles. This second series consisting of n triangles **2α** and n triangles $2\beta_1$ is inscribed in a second frustoconical ring with an angle of ($180° - 2\delta_1$) at the vertex.

Angle γ can be of any value or fixed at a whole fraction of 90°, let γ = 90/i, i being an integer.

The frustoconical rings succeed one another and, when γ = 90°/i, pass through a cylindrical ring (from which the iteration could also have started) until it becomes a flat ring after 4i-1 iterations. This first "sea urchin" with a height of **h** therefore consists of 4i (or 360°/γ) sets of **2n** triangles inscribed in a set comprising a flat ring and 4i-1 frustoconical rings.

Figure 3 shows a "sea urchin" for **n**=20, α=30°, and γ=15°. With a height of **h** and a diameter of **d**, it is inscribed in a cone with an angle **2ε** at the vertex. The iterations can continue in successive loops leading to the stacking of increasingly smaller "urchins", but (invariably when γ = 90°/i) of identical proportions (**h**/R_0=constant) up to **h** (and R_0) reduced to zero, inscribed in a cone with an angle at the vertex **2ε**, and height **H**.

Two other cones characterise the "sea urchin": the one whose base is the radius starting circle R_0 (angle at the vertex $2\varepsilon_1$) and the one whose base is the radius circle $R_{0.1}$ (angle at the vertex $2\varepsilon_2$). The aperture angles (**ε**, **ε1**, **ε2**), which envelop the polyhedron, thus depend on **n** (the number of the sides of the starting polygon), **2α** (the angle at the vertex of the isosceles triangles), and γ (the angle between the planes containing the first n triangles of the first series of **2n** triangles and the horizontal plane, or the "step", which becomes 2γ for the first n triangles of the second series of **2n** triangles…) corresponding to δ_0, δ_1… (the angles of the cones enveloping the successive series of **2n** triangles).

Figure 2

$$R_1 = R_0 . \left(1 - \frac{\sin N . \cos \gamma}{\tan \alpha}\right)$$

$$L_1 = R_1 . \sin N$$

$$R_2 = R_0 . \left(1 - \frac{\sin N . \cos \gamma}{\tan \alpha}\right) . \left(1 - \frac{\sin N . \cos 2\gamma}{\tan \alpha}\right)$$

$$L_2 = R_2 . \sin N$$

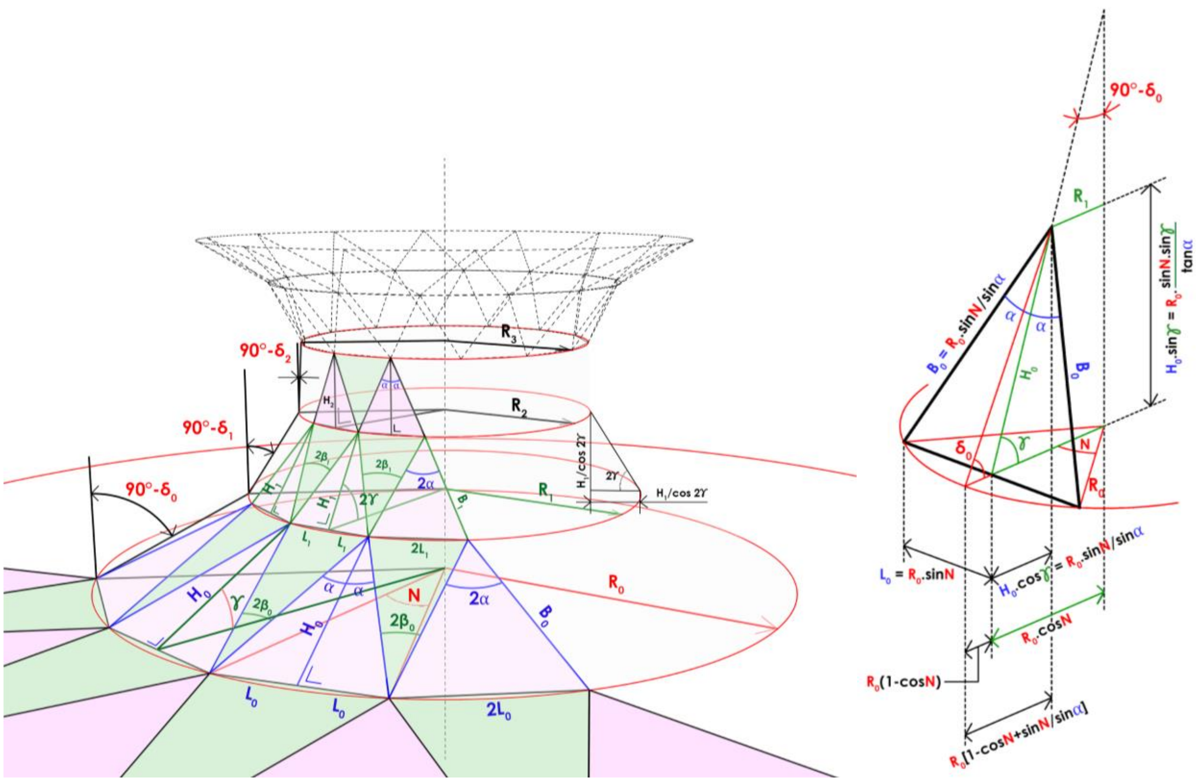
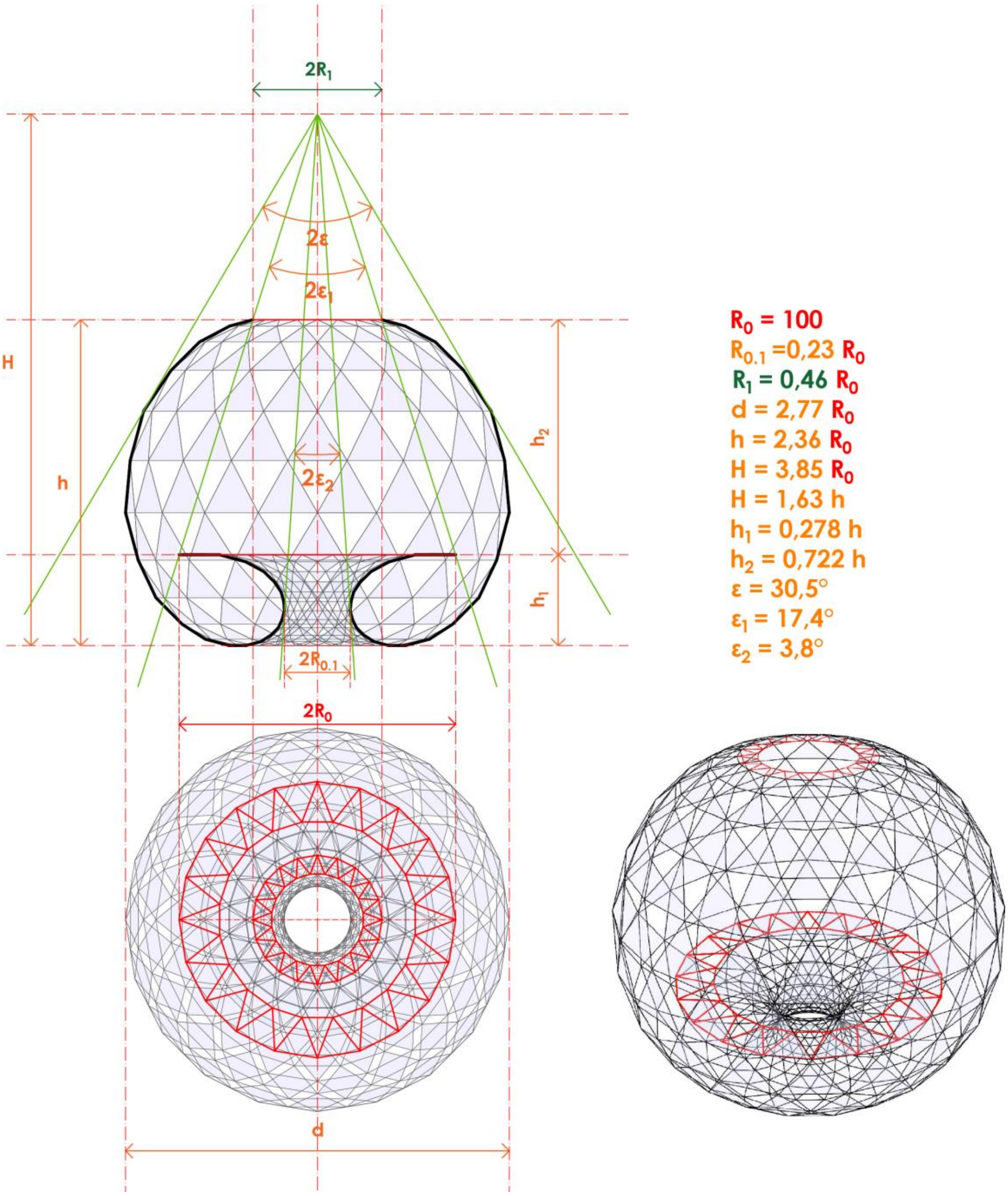


*Figure 3: "sea urchin" for **n**=20, α=30°, and γ=15°.*



1. Step "γ = 90°/I".

A polyhedron with **n** = 20 sides and equilateral triangles, with **2α** = 60°, are arbitrarily taken as the starting form. The values of γ, chosen to always go from 0 to 90 ° (i.e. γ = 90 ° / i, i being an integer), are respectively 45°; 30°; 15°; 10°; 7.5°; 5°; 2.5°; 1°. The starting triangles are placed either inside the initial polygon or outside. Figure 4 illustrates the first "sea urchins" corresponding to the first six values of γ, and figure 5 their stack (for the eight values of γ).

Figure 4: first "sea urchins" for **n** *= 20;* $\alpha = 30°$*;* $\gamma = 90°/i$*.*

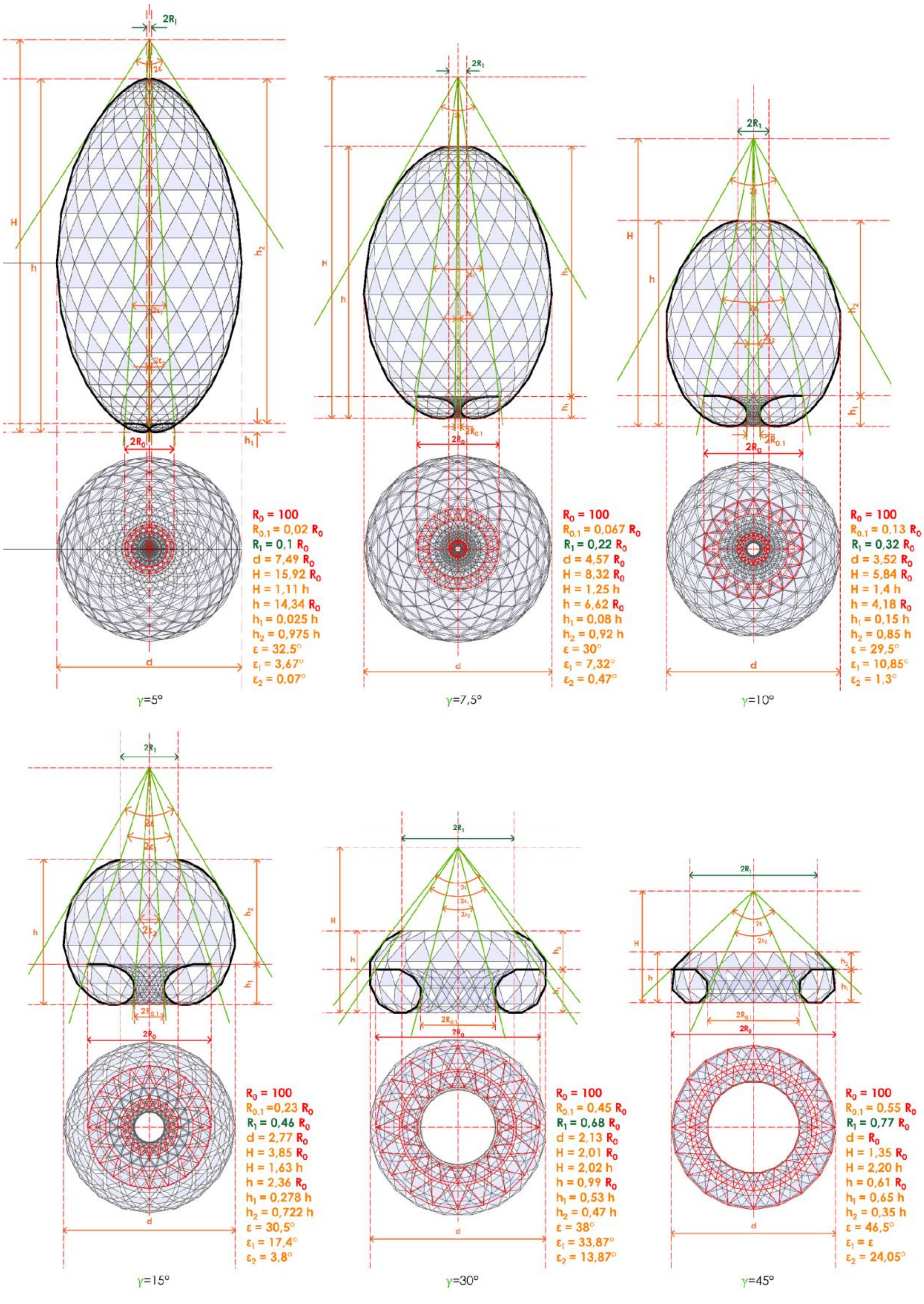
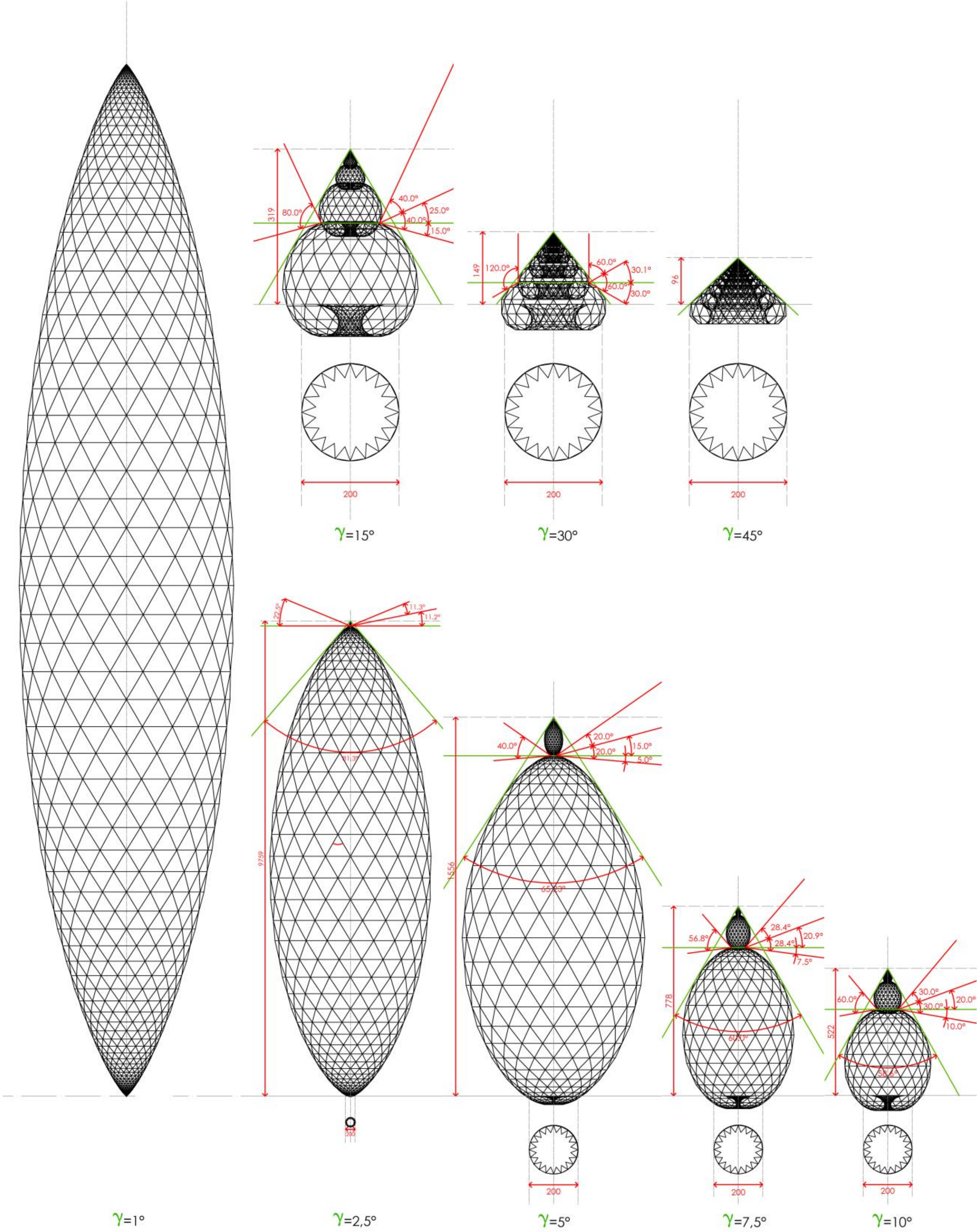


Figure 5: "sea urchin" stack for **n** *= 20;* α *= 30°;* γ *= 90°/i.*



2. Step "$\gamma \neq 90°/i$ ".

The general form does not change significantly when $\gamma \neq 90°/i$, as illustrated in figure 6 for γ = 7°, 11°, 13°, 17°, 17°, 19°, 23°, 29°, 31°, and 37° (the prime numbers which are not submultiples of 90°) with the four forms, in red, for γ = 5°, 10°, 15°, and 30° (in multiples of 90°) except that it is inscribed in a slightly curved cone with a negative Gaussian curvature. Figure 7 shows the starting sea urchins.

Figure 6

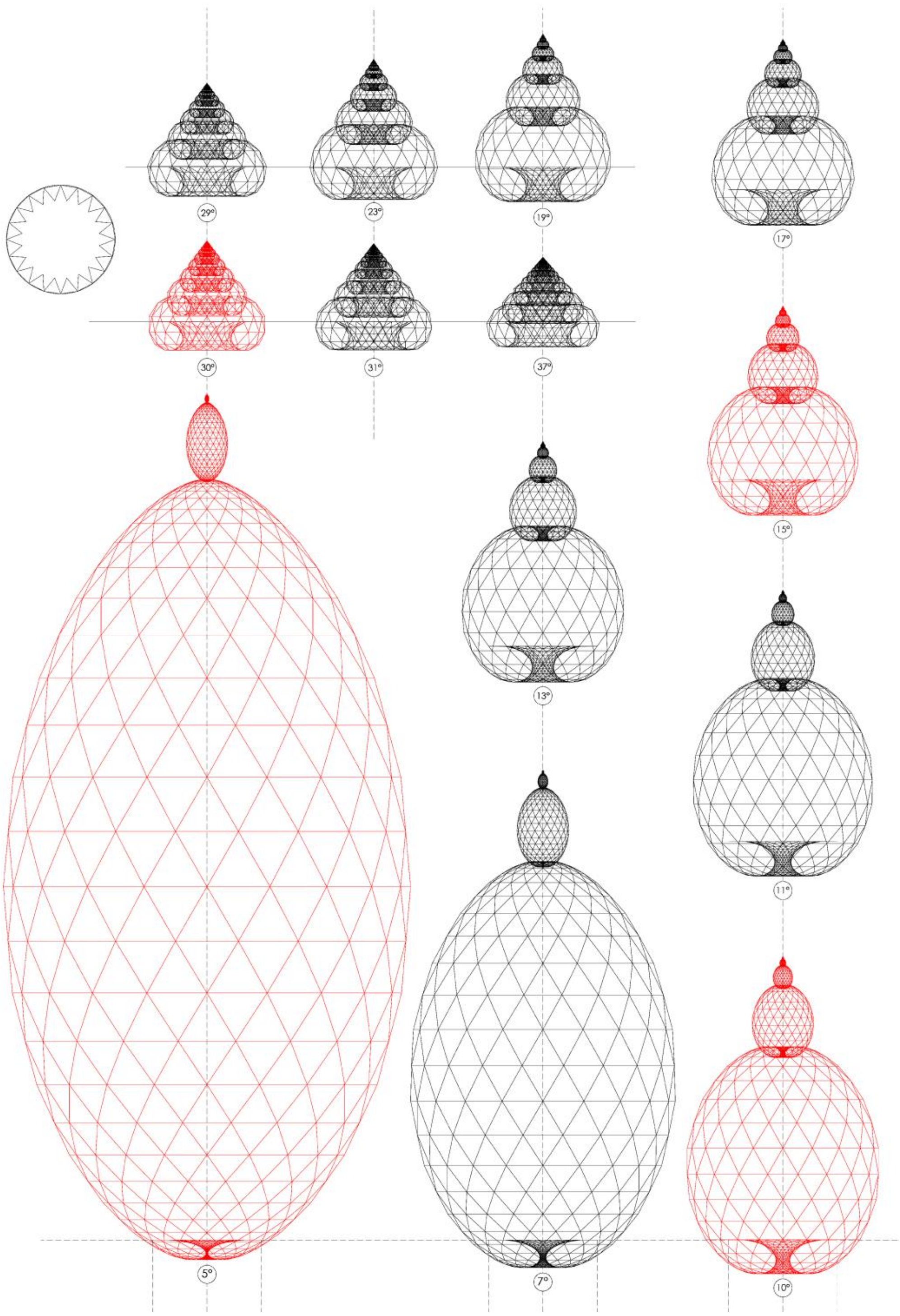
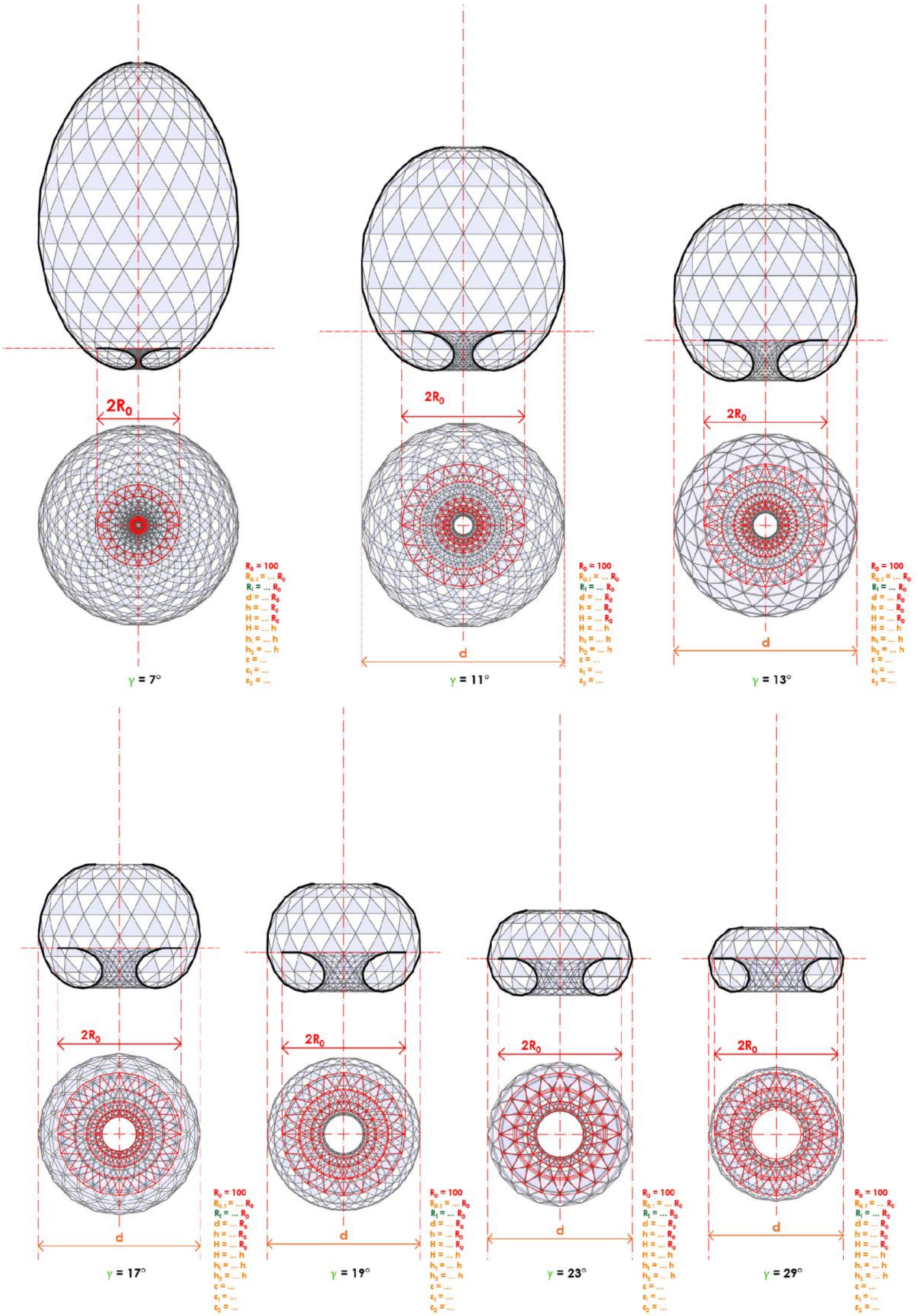


Figure 7: starting "sea urchins" for prime numbers.



3. The polyhedron for different "**n**".

The polyhedron is then drawn for values of **n** equal to 20, 30, 40, 50, 60, and 70 with α= *30° and* γ=5° (figure 8). Figure 9 shows the corresponding "sea urchins".

Figure 8: **n** *variable;* α *=30°;* γ = 5°

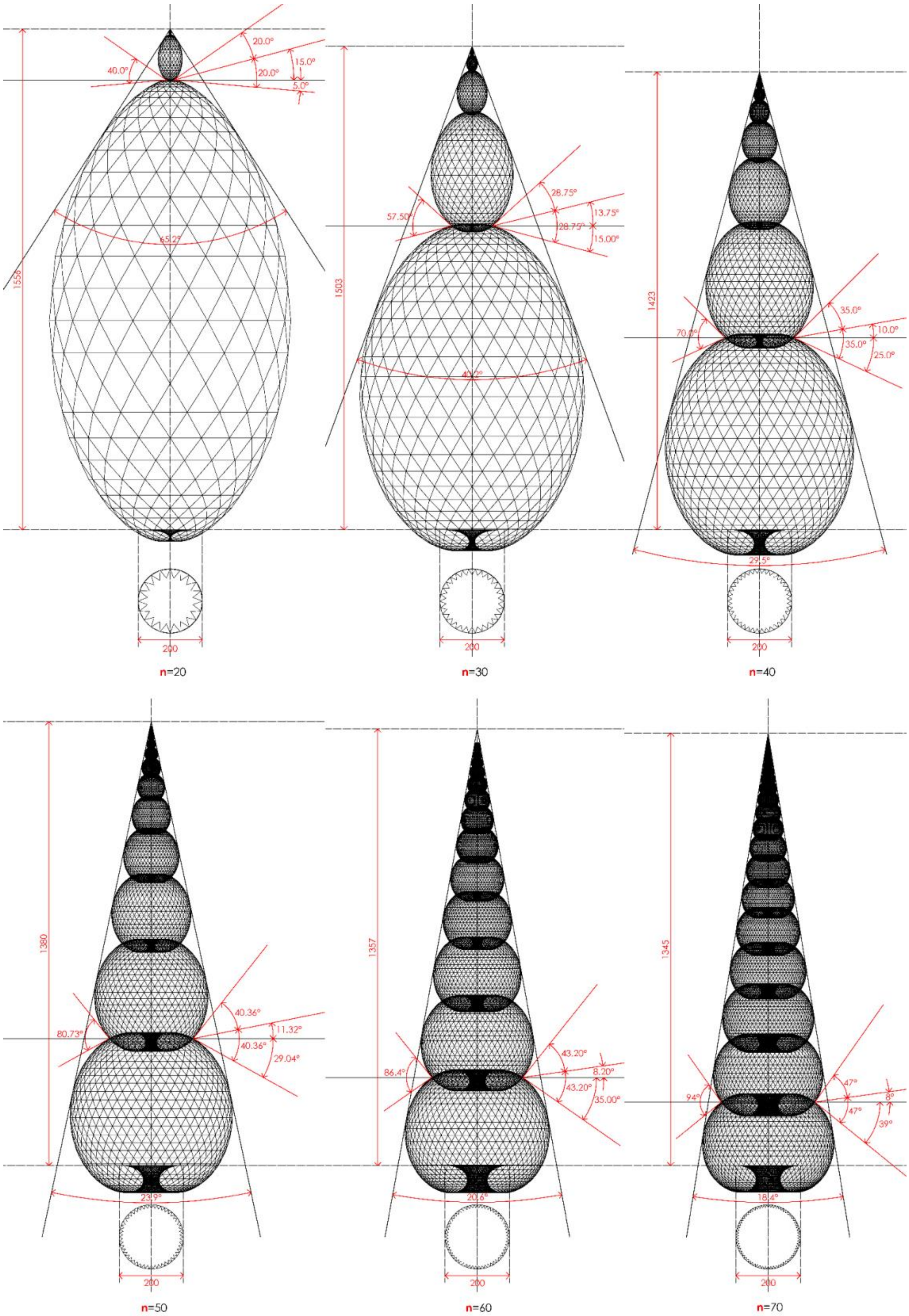
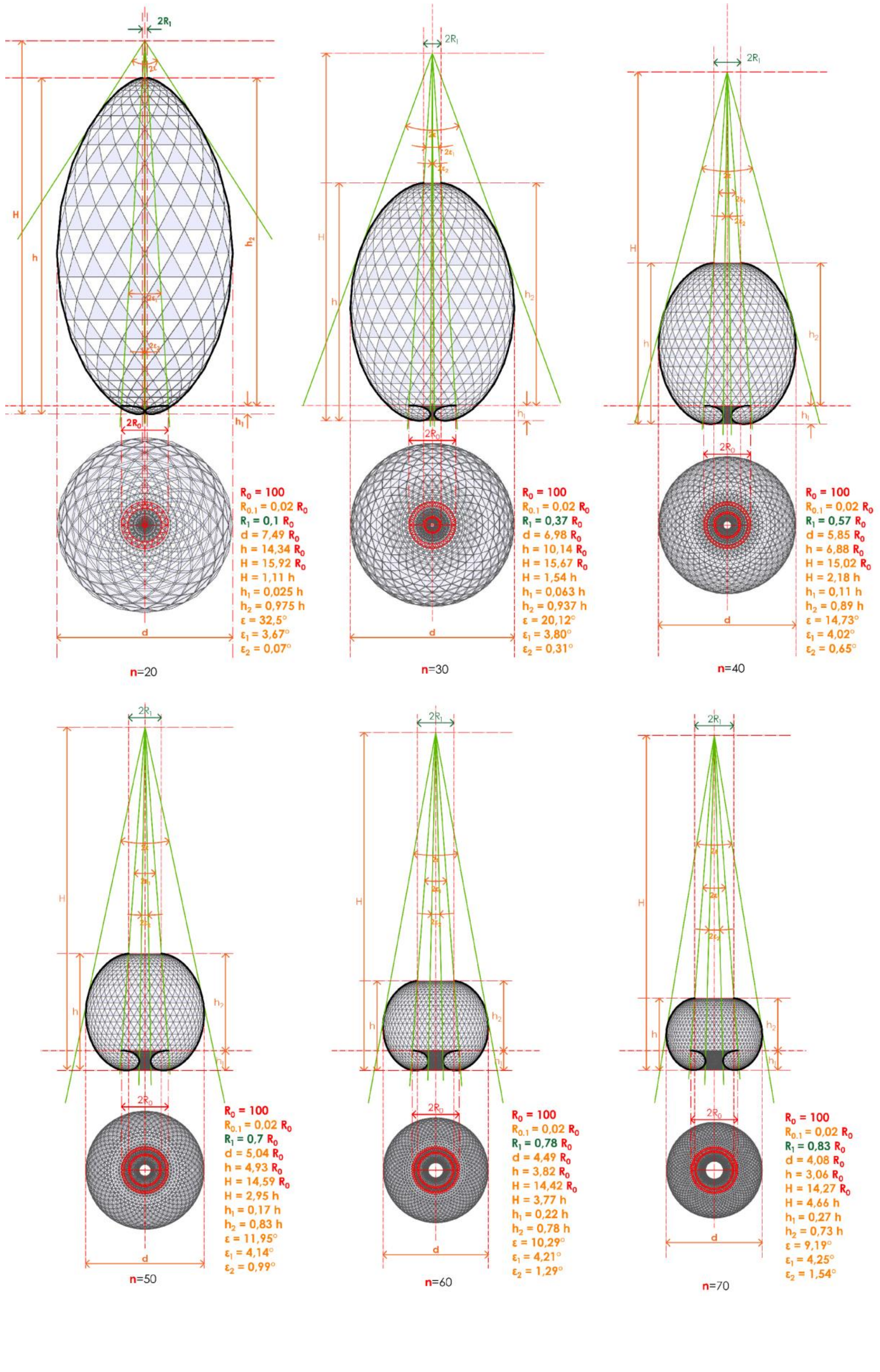


*Figure 9: "sea urchins" for **n** variable; $\alpha = 30°$; $\gamma = 5°$.*



4. Polyhedron for different angles α

The form can also be drawn for different α. It flattens when α increases, as shown in figure 10 for α = 45° (with n=20 and different γ) or, conversely, lengthens when α decreases.

*Figure 10: "sea urchin" stack for **n** = 20; α =45°; γ = 90°/i*